Reliability Modeling with the MIS Technique

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August 21, 2015

1 MIS Technique

The MIS technique constructs a Markov Chain where each transition between different states is caused by the failure of one component of the overall system. A system of n components will have $m \leq 2^n$ possible states. When $m = 2^n$, each state represents one of the possible permutations of failed components. If $m < 2^n$, one or more of the states is a superstate representing multiple permutations of failed components. State S_0 is the 'perfect' state where every component of the system is functional. We define state S_{m-1} to be the only state where the system has failed. S_{m-1} is an absorbing state, since reliability does not consider the ability of the system to recover from failure. States S_1 through S_{m-2} indicate that the system is functional, but degraded. All the states must be mutually exclusive and collectively exhaustive.

This technique determines system reliability by sequentially considering the effect of each component on the system's state distribution. This is represented as taking n steps through a Markov Chain having random variables $Y_0, \cdots, Y_n$ where state Y_l contains the state probability distribution after l steps. The transition probability matrix for each step, P_l, has elements

$$p_{ij}(l) = Pr(Y_l = S_j | Y_{l-1} = S_i)$$

In other words, given that the system consisting of components 1 through $l-1$ is in state S_j, the system consisting of components 1 through l will be in state S_i with probability $p_{ij}(l)$. It is this conditional probability that allows us to evaluate total system reliability based on component reliability: in general, $p_{ij}(l)$ is only dependent on the reliability of component l.

Note that since the failure state S_{m-1} is absorbing, $p_{m-1m-1}(l) = 1$ for all l.

The Chapman-Kolmogorov equation for DTMCs states that if $p_{ik}(s)$ is the probability of reaching state k from state i in s steps and $p_{kj}(t)$ is the probability of reaching state j from k in t steps, then

$$p_{ij}(s+t) = \sum_k p_{ik}(s) p_{kj}(t)$$

We can apply this equation to derive that the n-step transition probability matrix is $\prod_{l=1}^{n} P_l$ [1].

Because we are usually interested in the reliability of the perfect system, we define the initial state probability vector:

$$\Pi_0 = [1, 0, 0, \cdots, 0]^T$$

Finally, to calculate reliability, we need to sum the probabilities of being in each functional state in the system containing components 1 through n. (We can simply sum the probabilities of each state because they are mutually exclusive and collectively exhaustive.) We define which states are functional with the vector u where $u_i = 1$ if S_i is a functional state and $u_i = 0$ otherwise. Since S_{m-1} is the only failed state,

$$u = [1, 1, \cdots, 1, 0]$$

Putting all this together:

$$R = \Pi_0^T * \prod_{l=1}^{n} P_l * u$$

2 Detailed MIS Example

We will use the two-line example from the Reliability Metrics section of the paper [2], with line reliability $p_L = 1 - q_L$ and both lines in parallel (i.e. failure only if both lines fail).

We define the system states as shown in Table 1 (this is the $m = 2^n$ case).

Table 1: Binary Matrix

	Components	
States	l_1	l_2
S_0	1	1
S_1	1	0
S_2	0	1
S_3	0	0

$$\Pi_0 = [1, 0, 0, 0]^T$$

$$u = [1, 1, 1, 0]^T$$

We choose values for $p_{ij}(l)$ to reflect that functional components can remain functional or fail, but failed components cannot recover from failure:

$$P_{l_1} = \begin{bmatrix} p_L & 0 & q_L & 0 \\ 0 & p_L & 0 & q_L \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_{l_2} = \begin{bmatrix} p_L & q_L & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & p_L & q_L \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This gives

$$R = \Pi_0^T * P_{l_1} * P_{l_2} * u = p_L^2 + 2p_L q_L$$

We can draw the Markov Chain corresponding to this structure as shown in Figure 1.

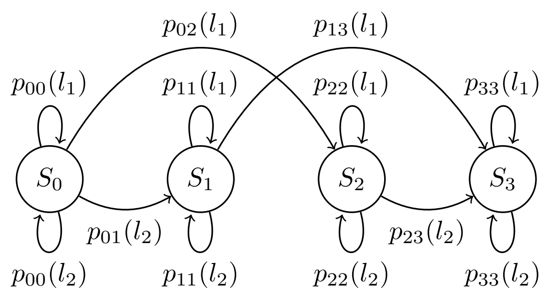

Figure 1: Two-Line MIS Markov Chain

3 IEEE 9-bus MIS Explanation

Let l_1, l_2, l_3 be the three lines that cause cascading failure when they fail, and l_4 - l_9 be the remaining lines.

In order to simplify the calculations, we will collect several system states into superstates as shown in Table 2 (the $m < 2^n$ case). S_0 is a fully functional system, S_1 is a system with one non-cascading line failure, S_2 is the unreachable case where one line that causes cascading failure fails without cascading, and S_3 is the failure state where more than one line has failed. Since S_2 is unreachable, we could remove it entirely from the calculations, but we will leave it in for completeness.

We let

$$u = [1, 1, 0, 0]^T$$

(Note that it does not matter if $u[2]$ is 1 or 0.)

Table 2: IEEE 9-bus states

		Components					
Superstates	States	l_1	l_2	l_3	l_4	$\cdots$	l_9
S_0	s_0	1	1	1	1	$\cdots$	1
S_1	s_1	1	1	1	0	$\cdots$	1
	$\vdots$						
	s_6	1	1	1	1	$\cdots$	0
S_2	s_7	0	1	1	1	$\cdots$	1
	s_8	1	0	1	1	$\cdots$	1
	s_9	1	1	0	1	$\cdots$	1
S_3	$\vdots$						

$$\Pi_0 = [1, 0, 0, 0]^T$$

Because we have combined several states into superstates, lines l_1, l_2, and l_3 share the same transition matrix, as do lines l_4 through l_9:

$$P_{l_{1,2,3}} = \begin{bmatrix} p_L & 0 & 0 & q_L \\ 0 & p_L & 0 & q_L \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_{l_{4-9}} = \begin{bmatrix} p_L & q_L & 0 & 0 \\ 0 & p_L & 0 & q_L \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Resulting in

$$R = \Pi_0^T * P_{l_{1,2,3}}^3 * P_{l_{4-9}}^6 * u = p_L^9 + 6p_L^8 q_L$$

References

[1] S. Chadjiconstantinidis and M. V. Koutras, "Measures of component importance for markov chain imbeddable reliability structures," *Naval Research Logistics (NRL)*, vol. 46, no. 6, pp. 613–639, 1999.

[2] M. N. Albasrawi, N. Jarus, K. A. Joshi, and S. S. Sarvestani, "Analysis of reliability and resilience for smart grids," in *Computer Software and Applications Conference (COMPSAC), 2014 IEEE 38th Annual*. IEEE, 2014, pp. 529–534.

[3] W. Kuo and M. J. Zuo, *Optimal reliability modeling: principles and applications*. John Wiley & Sons, 2003.

[4] L. Xie and M. D. Ilic, "Module-based modeling of cyber-physical power systems." IEEE, Jun. 2008, pp. 513–518.

[5] M. V. Boutsikas and M. V. Koutras, "Reliability approximation for markov chain imbeddable systems," *Methodology and Computing in Applied Probability*, vol. 2, no. 4, pp. 393–411, 2000.

[6] K. S. Trivedi, *Probability & statistics with reliability, queuing and computer science applications*. John Wiley & Sons, 2008.